

**Math 332-001 • December 15, 2009**

Final Examination • Victor Matveev

This is a closed-book exam; neither notes nor calculators are allowed.

**Please choose between problems 2 and 2'**

1) (18pts) For each, find **all** distinct values of  $z$  in Cartesian form, and show them as points in the  $z$ -plane:

(a)  $z = |(1+i)^i|$       (b)  $\sinh z = 0$       (c)  $z = (1-\sqrt{3}i)^{3/2}$

2) (12pts) Find and sketch the two curves that result from the mapping of a vertical line  $\operatorname{Re} z=1$  and a horizontal line  $\operatorname{Im} z=1$  under the transformation  $w = e^{iz}$

2') (12pts) Which function does the series  $\sum_{n=0}^{\infty} \frac{(2+i)^n}{(z+i)^{n+1}}$  converge to? Sketch the region where this series converges.

3) (12pts) Is the function  $f = iz + |z|^2$  differentiable anywhere? Use the Cauchy-Riemann equations to answer this question.

4) (16pts) Describe the singularity type and find the residue (you will need only two terms in each series) for the following functions at the singular point  $z=0$ :

(a)  $f(z) = \frac{\sin(2z) \sinh(z)}{z^5}$       (b)  $z^3 \cos\left(\frac{1}{z^2}\right)$

If you can't recall the series for  $\sinh z$ , you can easily derive the first few terms.

5) (21pts) Describe all singularities of the integrand inside the integration contour, and calculate each integral, using the method of residues where appropriate, or other methods. Each integration contour is a circle of specified radius:

(a)  $\oint_{|z|=3} \frac{e^z dz}{z^2 + 4}$       (b)  $\oint_{|z|=1} \frac{z}{1 - \cos z} dz$       (c)  $\oint_{|z|=4} \frac{dz}{z^{1/2}}$

Note: in (c), use the principal value of  $z^{1/2}$

6) (21pts) Calculate **any two** of the following three integrals, **explaining each step**.

(a)  $\int_0^{2\pi} \frac{d\theta}{3 + \sin \theta}$       (b)  $\int_0^{\infty} \frac{x^3 \sin(3x)}{x^4 + 4} dx$       (c)  $\int_0^{\infty} \frac{\sin(2x)}{x(x^2 + 1)} dx$

If problem (6c), use the following contour:

